

Task 1 (50 points)

The first exam is coming up on 20–OCT–2026. Please review all distributed lecture notes, your own lecture notes, and all supplementary notes from the PLCs, and write a “cheat sheet” with the most important formulas encountered so far in the lecture. Please also summarize the most important concepts encountered so far.

Task 2 (80 points = four times 20 points)

We investigate the harmonic oscillator using Green function techniques and apply the Green function formalism to the harmonic oscillator. The “displacement” $x = x(t)$ of a damped, harmonic oscillator satisfies the equation

$$\ddot{x} + \gamma \dot{x} + \omega_0^2 x = f(t), \quad (1)$$

where $f(t)$ is the external force, normalized by the particle’s mass.

(a) Recall, from the lecture notes, all steps necessary to obtain the Green function $g = g(t - t')$ for this equation. Note that it has to fulfill the equation

$$\ddot{g}(t - t') + \gamma \dot{g}(t - t') + \omega_0^2 g(t - t') = \delta(t - t'). \quad (2)$$

(b) Why is this Green function naturally obtained as the retarded Green function? Why do you have to change the sign of the damping term in order to obtain the advanced Green function? Consider the effect of time reversal on the equation of motion, and provide an illustrative discussion.

(c) Assume that the “force” is given by $f(t) = v_0 \delta(t)$, where v_0 has the dimension of a velocity. Under the boundary conditions $x(-\infty) = 0$, and $\dot{x}(-\infty) = 0$, evaluate $x(t)$.

(d) Assume that the “force” is given by $f(t) = \Theta(-t) \exp(t/\tau)$, with vanishing boundary conditions in the infinite past, $x(-\infty) = 0$, and $\dot{x}(-\infty) = 0$. Possibly with the help of computer algebra, or, with the help of pertinent lecture notes (please read this task carefully, there is only a single force term, not two), evaluate $x(t)$.

You may utilize the hints given during the lecture, but please formulate your thoughts carefully and comprehensively, so that it becomes clear that you show all your work.

Task 3 (50 points)

Review, and recall, all intermediate steps leading to the generalized Drude model of the dielectric function, given in Eqs. (2.222) and (2.223) of the lecture notes,

$$\epsilon(\omega) = \epsilon_0 \left[1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2 - i\omega\gamma} \right], \quad (3)$$

where we have identified the plasma frequency ω_p as follows,

$$\omega_p^2 = \frac{N_V e^2}{\epsilon_0 m_e}. \quad (4)$$

Explain all symbols, as defined in the lecture notes, and explain why the model is relevant for a collection of electrons with number density N_V , bound in harmonic potentials with resonant frequency ω_0 .

The tasks are due Thursday, 15–OCT–2026, with a possible extension to Friday, 16–OCT–2026.