

Task 1 (40 points)Consider the functions $F(t)$

$$F(t) = \frac{1}{b} \exp\left(-\frac{(t-t_0)^2}{a^2}\right), \quad (1)$$

Assume that a and b are real.(a.) Calculate $\|F\|$. Express b in terms of a so that F is normalized to unity.(b.) Calculate $\langle t \rangle$ and $\langle t^2 \rangle$.(c.) Calculate the Fourier transform $\tilde{F}(\omega)$.(d.) Calculate $\langle \omega \rangle$ and $\langle \omega^2 \rangle$.(e.) Verify that the relation $\Delta t \cdot \Delta \omega \gtrsim \frac{1}{2}$ is fulfilled for the given example.**Task 2** (50 points)

Derive the expression

$$\begin{aligned} G_A(\vec{r} - \vec{r}', t - t') &= \frac{c}{4\pi\epsilon_0} \frac{1}{|\vec{r} - \vec{r}'|} \Theta(t' - t) \left\{ \delta\left(|\vec{r} - \vec{r}'| - c(t' - t)\right) - \delta\left(|\vec{r} - \vec{r}'| + c(t' - t)\right) \right\} \\ &= \frac{c}{2\pi\epsilon_0} \Theta(t' - t) \delta\left((\vec{r} - \vec{r}')^2 - c^2(t - t')^2\right) \end{aligned} \quad (2)$$

for the advanced Green function, by following the integrations given in the lecture for the retarded Green function, and using the “advanced” integration contour. **Hint:** Consider, first, the function P_{CA} defined in analogy to the function P_{CR} in the lecture notes.

Task 3 (20 points)

Consider the function

$$f(z) = \frac{\sin(z)}{z^6 \cos(z)} = \frac{\tan(z)}{z^6}. \quad (3)$$

Convince yourself that this function has a fifth-order pole at $z = 0$. Show that (you may use computer algebra but you must always include a printout of what you did)

$$\frac{1}{5!} \frac{d^5}{dz^5} \left\{ z^6 \left(\frac{\sin(z)}{z^6 \cos(z)} \right) \right\} \Big|_{z=0} = \frac{2}{15}. \quad (4)$$

Thus, calculate the residue

$$\text{Res}_{z=0} [\tan(z) z^{-6}]. \quad (5)$$

The tasks are due Thursday, 08-OCT-2026.