Task 1 (20 points)
(a) Consider a function of the form $W(\vec{r}, t) = f(kx - \omega\, t)$, where x is the x coordinate, k is wave vector, t is the time, and ω is the angular frequency. Show that $W(\vec{r}, t)$ satisfies the homogeneous form of the scalar wave equation.

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \vec{\nabla}^2 \right) W\,(\vec{r}, t) = 0\,, \tag{1}$$

provided $\omega/k = c$.

(b) Show that the same property holds for a function

$$W(\vec{r}, t) = f(\vec{k} \cdot \vec{r} - \omega t) = f(k_x\, x + k_y\, y + k_z z - \omega\, t), \tag{2}$$

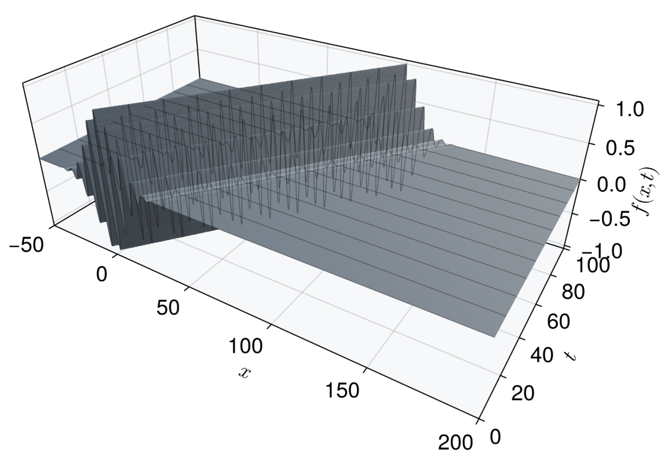
provided that the parameters ω and k fulfill the relationship $\omega/\sqrt{k_x^2 + k_y^2 + k_z^2} = c$.

Here, f can be any function, e.g., $f(\xi) = a\, \cos(b\, \xi)$ with constant a and b, or $f(\xi) = c\, \exp(-d\, \xi^2)$ with constant c and d.

Task 2 (40 points)
(a) Consider the function

$$W(x, t) = A\, \exp\left[-\frac{(kx - \omega t)^2}{b} \right] \cos(kx - \omega t) \tag{3}$$

as a mathematical model problem, with $k = \omega = 1$, $b = 200$ and $A = 1$, . Plot this function, using the computer symbolic language of your choice, in the range $x \in (-50, 200)$ and $t \in (0, 100)$ and convince yourself that the resulting image is consistent with a propagation of the wave train with unit velocity in the positive x direction. You should obtain an image similar to



(b) Now, relieve the restrictin $k = \omega = 1$. Calculate the wave vector k, the angular frequency ω, as well as the wavelength λ and the frequency ν, if the wave train in task **(a)** represents the field (electric and/or magnetic, it does not matter) of a photon of energy $E = 1.5$ eV. What is the unit of ω, compared to the unit of ν?

The tasks are due Thursday, 10–SEP–2026.