Physics/6211 **Electrodynamics II** **Dr. Ulrich Jentschura** **Spring Semester 2026**

Missouri S & T (Rolla, Missouri) **Exercise Sheet #1** **25–AUG–2026**

The current exercises need to be done without the help from any artificial-intelligence tools. In general, a recap of a number of basic mathematical concepts is indicated. The questions on this, current, exercise sheet should provide inspiration for this endeavor. The course on electrodynamics II carries certain prerequisites, and we have to make sure these are met. It will important for you to do a thorough recap of all material from previous courses on electrodynamics you have taken before, including Physics 6111 (Graduate Electrodynamics I). Also, any material on calculus, including vector analysis, will be needed.

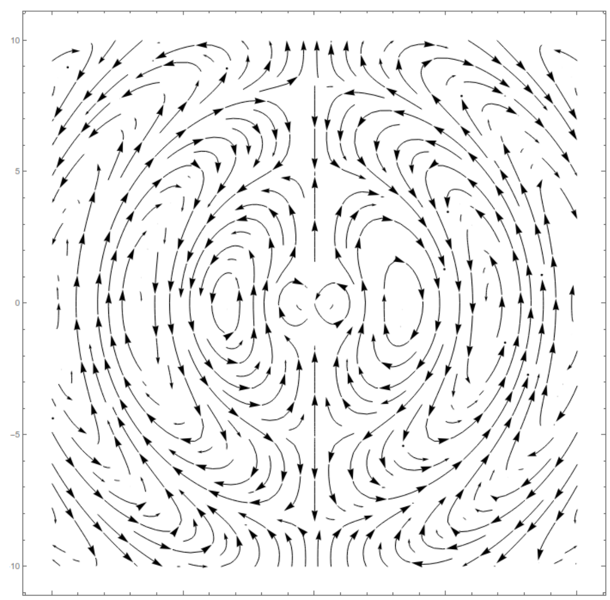
Task 1 (70 points)
Review, for yourself, the concepts of **(a)** the divergence theorem, **(b)** oriented surfaces and the right-hand rule, **(c)** Stokes's theorem, **(d)** Taylor expansions in three dimensions, **(e)** the longitudinal and transverse components of a vector field, **(f)** the Poisson equation, **(g)** the Laplace equation.

Write a mini-essay on each topic. For Stokes's theorem, discuss the concept of the orientation of a surface embedded in three-dimensional space, and the right-hand rule.

Task 2 (30 points)
Look at the following picture of the electric field lines of an oscillating (radiating) dipole, at a reference time during an oscillation period. Answer the following questions:

(i) Is the displayed electric field curl-free? If not, how can you tell?
Hint: Think of the integral form of Stokes's theorem and look at the field lines!

(ii) Use Faraday's Law to infer the direction of the magnetic field at a point of your choice in the figure.
Hint: Think of Faraday's law!



Task 3 (30 points)
(a) Use the parameterization $\vec{f} = \vec{f}(t_1, t_2)$ for a two-dimensional surface embedded in three-dimensional space. Show that the vector-valued infinitesimal surface element $\mathrm{d}\vec{A}$ can be calculated as

$$\mathrm{d}\vec{A} = \left(\frac{\partial \vec{f}}{\partial t_1} \times \frac{\partial \vec{f}}{\partial t_2} \right) \mathrm{d}t_1 \, \mathrm{d}t_2 \,. \tag{1}$$

Here,

$$\mathrm{d}\vec{A} = \widehat{n} \, \mathrm{d}A \tag{2}$$

is normal to the surface, and $\mathrm{d}A = |\mathrm{d}\vec{A}|$ is the modulus of the infinitesimal surface area element. **(b)** What physical dimension does $\mathrm{d}A$ have? **(c)** Use the parameterization $(t_1 = \theta,\, t_2 = \varphi)$

$$\vec{f}(\theta, \varphi) = \hat{e}_x\, R\, \sin\theta\, \cos\varphi + \hat{e}_y\, R\, \sin\theta\, \sin\varphi + \hat{e}_z\, R\, \cos\theta\,. \tag{3}$$

Characterize the surface represented by the above parameterization and show that

$$\mathrm{d}\vec{A} = \left(\frac{\partial \vec{f}}{\partial \theta} \times \frac{\partial \vec{f}}{\partial \varphi} \right) \mathrm{d}\theta\, \mathrm{d}\varphi = \hat{e}_r\, R^2\, \sin\theta\, \mathrm{d}\theta\, \mathrm{d}\varphi\,, \tag{4}$$

under a suitable identification (which one?) of $\hat{e}_r$. **(d)** What surface is described by the parameterization $(t_1 = \theta,\, t_2 = \varphi)$

$$\vec{f}(\theta, \varphi) = \hat{e}_x\, a\, \sin\theta\, \cos\varphi + \hat{e}_y\, b\, \sin\theta\, \sin\varphi + \hat{e}_z\, c\, \cos\theta\,, \tag{5}$$

with $a = 1.4\,\mathrm{m}$, $b = 0.4\,\mathrm{m}$, and $c = 0.2\,\mathrm{m}$? Calculate the **tangent plane** to the above surface at the point $\theta = \theta_0 = \pi/16$ and $\varphi = \varphi_0 = 0$, by finding a representation of the tangent plane in terms of two suitable variables. **Draw a schematic picture illustrating the geometric situation and the location of the tangent plane at the point with the parameterization $\theta = \theta_0 = \pi/16$ and $\varphi = \varphi_0 = 0$.**

Task 4 (30 points)
What is the motivation for the following "physics" sign conventions (for the arguments of the exponentials) and the normalization factors [*i.e*, the $1/(2\pi)^3$ prefactor for the Fourier backtransform] in the Fourier transform from coordinate space ($\vec{r}$) to wave-vector space ($\vec{k}$)? One defines

$$f(\vec{k}) = \int \mathrm{d}^3 r\, \exp(-\mathrm{i}\vec{k}\cdot\vec{r})\, f(\vec{r})\,, \qquad f(\vec{r}) = \int \frac{\mathrm{d}^3 k}{(2\pi)^3}\, \exp(\mathrm{i}\vec{k}\cdot\vec{r})\, f(\vec{k})\,. \tag{6}$$

Why do physicists use the above sign and normalization conventions?
Hint: Think of the momentum operator in quantum mechanics!

Task 5 (60 points)
(a) Show that a solution of the Poisson equation $\vec{\nabla}^2 \Phi(\vec{r}) = -\frac{1}{\epsilon_0}\, \rho(\vec{r})$ is given as

$$\Phi(\vec{r}) = \frac{1}{\epsilon_0} \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \frac{1}{\vec{k}^2}\, \exp(\mathrm{i}\vec{k}\cdot\vec{r})\, \rho(\vec{k})\,, \tag{7}$$

where $\rho(\vec{k})$ is the Fourier transform of $\rho(\vec{r})$.
(b) Show, using a method of your choice, that the Green's function of the three-dimensional Poisson equation (up to a solution of the homogeneous equation) is

$$g(\vec{r} - \vec{r}\,') = -\frac{1}{4\pi|\vec{r} - \vec{r}\,'|}\,, \qquad \vec{\nabla}^2\, g(\vec{r} - \vec{r}\,') = \delta^{(3)}(\vec{r} - \vec{r}\,')\,. \tag{8}$$

In particular, calculate (with derivation) the expression $\vec{\nabla}^2 g(\vec{r} - \vec{r}\,')$.
(c) Show that the formula $\Phi(\vec{r}) = -\frac{1}{\epsilon_0} \int \mathrm{d}^3 r'\, g(\vec{r} - \vec{r}\,')\, \rho(\vec{r}\,')$ is equivalent to Eq. (7).
Hint: Express $g(\vec{r} - \vec{r}\,')$ as a Fourier transform of [something].

(d) Why are the electric fields encountered in electrodynamics I (electrostatics) curl-free?
Hint: Consider Faraday's law.

(e) Write Gauss's law, $\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0$, in terms of the transverse and longitudinal components of the electric field, and simplify.
Hint: Consider the decomposition of any electric field (any vector field) into transverse and longitudinal components.

The tasks are due Thursday, 03–SEP–2026. Have fun doing the problems!