Task 1 (Unmarked) (Non–Uniform Convergence) Investigate the potential given in Eq. (6.75) of the lecture notes,

$$\Phi\left(x,y\right) = \frac{4\Phi_0}{\pi} \sum_{n=0}^{\infty} \frac{\sin\left[\left(2n+1\right)\pi x/a\right]}{2n+1} \, \frac{\sinh\left[\left(2n+1\right)\pi y/a\right]}{\sinh\left[\left(2n+1\right)\pi b/a\right]} \,. \tag{1}$$

Show the following properties which illustrate the non-uniform convergence at the point $x = 0$, and $y = b$,

$$\lim_{\epsilon \to 0^+} \Phi(0, b-\epsilon) = 0 \,, \qquad \lim_{\epsilon \to 0^+} \Phi(\epsilon, b) = \Phi_0 \,. \tag{2}$$

Task 2 (Unmarked) (Laplace Equation and Potential) Investigate the potential given in Eq. (6.90) of the lecture notes,

$$\Phi(x,y) = \frac{V_0}{\mathrm{i}\pi} \, \ln\left(\frac{\mathrm{e}^{\mathrm{i}\pi y/b} + \mathrm{e}^{\pi x/b}}{\mathrm{e}^{-\mathrm{i}\pi y/b} + \mathrm{e}^{\pi x/b}}\right) \,. \tag{3}$$

Show that, for real and positive $x > 0$, and $0 < y < b$, the result for $\Phi(x,y)$ is real rather than complex. Then, plot the potential given in Eq. (3), as a function of x and y, for your preferred choice of the parameter b, over a suitable range of x and y variables.

Task 3 (Unmarked) (Non–Uniform Convergence) Investigate, once more, Eq. (6.90) of the lecture notes, which is given above in Eq. (3). Show that

$$\lim_{\epsilon \to 0^+} \Phi(\epsilon, b) = 0 \,. \tag{4}$$

Furthermore, show that

$$\lim_{\epsilon \to 0^+} \Phi(0, b-\epsilon) = V_0 \,. \tag{5}$$

For the latter relation, use the result

$$\lim_{\eta \to 0^+} \ln(-1 \pm \mathrm{i}\eta) = \pm \mathrm{i}\pi \,. \tag{6}$$

Show that, as you calculate $\lim_{\epsilon \to 0^+} \Phi(0, b-\epsilon)$, you obtain an expression of the type given in Eq. (6), where the imaginary part of the argument of the logarithm is positive.

The tasks are given as voluntary exercises for the holidays.