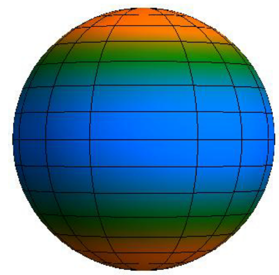
Task 1 (50 points)
Calculate the electrostatic potential outside of the charge distribution

$$\rho(\vec{r}) = \frac{5Q}{a^2} \left(3\cos^2\theta - 1\right) \delta(r-a)\,. \tag{1}$$

Why does $\rho(\vec{r})$ have the correct physical dimension of charge per volume, as required for a charge distribution? You may use the fact that the angular dependence can be described in terms of a Y_{20} function,

$$Y_{20}(\theta,\phi) = \frac{1}{4}\sqrt{\frac{5}{\pi}} \left(3\cos^2\theta - 1\right) . \tag{2}$$



If red areas mark positive charge, blue areas denote negative charge, and neutral areas are green, then the sphere looks like the one given on the right.
Hint: Your result should look similar to

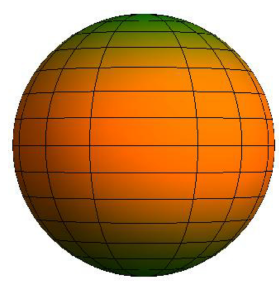
$$\Phi(\vec{r}) = \frac{Qa^2}{\epsilon_0\, r^3} \left(3\cos^2\theta - 1\right) , \qquad r > a\,. \tag{3}$$

Task 2 (50 points)
Calculate the multipole decomposition into monopole, dipole and quadrupole moments for the charge distribution

$$\rho(\vec{r}) = \frac{Q}{a^2} \sin\theta\, \delta(r-a)\,. \tag{4}$$

Evaluate all multipoles of the given charge distribution with angular momenta $\ell = 0, 1, 2$ and write down the final potential. If red areas mark positive charge, blue areas denote negative charge, and neutral areas are green, then the sphere looks like the one on the right.
Hint: The potential should finally be obtained as follows,

$$\Phi(\vec{r}) = \frac{\pi\, Q}{4\,\epsilon_0\, r} - \frac{\pi\, Q\, a^2}{64\,\epsilon_0\, r^3} \left(3\,\cos^2\theta - 1\right) . \tag{5}$$



Task 3 (50 points)
Perform the multipole decomposition of the electrostatic potential for the charge distribution

$$\rho(\vec{r}) = \frac{Q}{a^2} \left(5\cos^2\theta - 1\right) \sin\theta\, \cos\varphi\, \delta(r-a)\,. \tag{6}$$

Hint: Write $\cos\varphi$ in terms of exponential and compare with given forms of particular spherical harmonic in the lecture notes. Show all intermediate steps of your calculations.

The tasks are due Thursday, 07–MAY–2026.