Task 1 (30 points)
Consider a uniformly charged sphere with surface charge density σ_0, centered at the origin. Show, by analogy with a uniformly charged plane, that the corresponding volume charge density is

$$\rho(\vec{r}) = \sigma_0 \, \delta(r - a) \,. \tag{1}$$

Then, integrate over all space and show that you obtain the total charge of the sphere $a = 4\pi a^2 \sigma_0$. You can be inspired by lecture notes.

Task 2 (60 points) (Challenging!)
Consider a surface defined in three-dimensional space by the equation

$$F(\vec{r}) = F(x, y, z) = 0 \,. \tag{2}$$

Let $\sigma = \sigma(\vec{r})$ be a variable (position-dependent) surface charge density.

(a) Show that the corresponding volume charge density is

$$\rho(\vec{r}) = \sigma(\vec{r}) \, \delta(F(\vec{r})) \, |\vec{\nabla} F(\vec{r})| \,. \tag{3}$$

Hint: Consider the fact that the gradient of the function F will always point perpendicularly away from the surface defined by the equation $F = F(\vec{r}) = 0$.

(b) Then, show the compatibility of the result given in Eq. (3) with Eq. (1), by considering the defining equation of the sphere ($F(\vec{r}) = |\vec{r}| - a = 0$).

Task 3 (60 points) (Challenging!)
While our course is focused on electrostatics, a brief recap on basic aspects of electrodynamics cannot hurt. Consider a situation with

$$\rho(\vec{r}, t) = 0 \,, \qquad \vec{J}(\vec{r}, t) = \vec{0} \,. \tag{4}$$

(a) Assume that

$$\vec{E} = \vec{E}(\vec{r}, t) = E_0 \, \hat{\mathrm{e}}_z \, \cos(kx - \omega t) \tag{5}$$

From the Maxwell equations, find $\vec{B} = \vec{B}(\vec{r}, t)$ so that the resulting field configuration describes a travel plane wave.

(b) Find an expression for the Poynting vector

$$\vec{S} = \frac{1}{\mu_0} \left[\vec{E}(\vec{r}, t) \times \vec{B}(\vec{r}, t) \right] \,. \tag{6}$$

(b) Find the time-averaged integral P (averaged over a laser period), where

$$P = \left\langle \int_{\delta A} \mathrm{d}\vec{A} \cdot \vec{S}(\vec{r}, t) \right\rangle \,, \tag{7}$$

where δA is a rectangular area of magnitude $\delta y \, \delta z$, located in the yz plane (*i.e.*, the plane defined by the equation $x = 0$), with corners

$$\vec{r}_1 = -(\delta y/2) \, \hat{\mathrm{e}}_y - (\delta z/2) \, \hat{\mathrm{e}}_z \,, \tag{8}$$

$$\vec{r}_2 = (\delta y/2) \, \hat{\mathrm{e}}_y - (\delta z/2) \, \hat{\mathrm{e}}_z \,, \tag{9}$$

$$\vec{r}_3 = (\delta y/2) \, \hat{\mathrm{e}}_y + (\delta z/2) \, \hat{\mathrm{e}}_z \,, \tag{10}$$

$$\vec{r}_4 = -(\delta y/2) \, \hat{\mathrm{e}}_y + (\delta z/2) \, \hat{\mathrm{e}}_z \,, \tag{11}$$

Convince yourself, by dimensional analysis, that P has the correct dimension (Joule per second).

The tasks are due Thursday, 23-APR-2026. Have fun doing the problems!