Task 1 (60 points)
Consider the integral form of the relation that connects a charge distribution $\rho = \rho(\vec{r}\,')$ to a scalar potential $\Phi(\vec{r})$,

$$\Phi(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \mathrm{d}^3 r' \, \frac{1}{|\vec{r} - \vec{r}\,'|} \, \rho(\vec{r}\,') \,, \tag{1}$$

and the discrete-sum formula (for a collection of N point charges),

$$\Phi(\vec{r}) = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{N} \frac{q_i}{|\vec{r} - \vec{r}_i|} \,. \tag{2}$$

Use a suitable *ansatz* for the charge density $\rho = \rho(\vec{r}\,')$ for the given collection of point charges, to show how to transform Eq. (1) into Eq. (2). Then, show how to derive Eq. (1) as a limiting case of Eq. (2), in the limit of an infinite collection ($N \to \infty$) of point charges located at point $\vec{r}_i$. **You might be inspired by lecture notes (regarding corresponding expressions for the electric field, not the scalar potential). When calculating the expressions, carefully distiguish independent arguments of functions, integration variables, and parameters!!!**

Task 2 (60 points)
Now, consider the Poisson equation (in three dimensions) for a charge distribution composed of **three** point charges,

$$\vec{\nabla}^2 \, \Phi(\vec{r}) = -\frac{1}{\epsilon_0} \, \rho(\vec{r}) \,, \qquad \rho(\vec{r}) = q_1 \, \delta^{(3)}(\vec{r} - \vec{r}_1) + q_2 \, \delta^{(3)}(\vec{r} - \vec{r}_2) + q_3 \, \delta^{(3)}(\vec{r} - \vec{r}_3) \,. \tag{3}$$

Symbols are defined as in the lecture. The three point charges are places at $\vec{r}_1$, $\vec{r}_2$ and $\vec{r}_3$.

(a.) Using the Green function of the three-dimensional Poisson equation, calculate the **electrostatic potential** $\Phi(\vec{r})$ and the electric field $\vec{E}(\vec{r})$,

$$\Phi(\vec{r}) = -\frac{1}{\epsilon_0} \int \mathrm{d}^3 r' \, g(\vec{r} - \vec{r}\,') \, \rho(\vec{r}\,') \,, \qquad \vec{E}(\vec{r}) = -\vec{\nabla}\Phi(\vec{r}) \,, \tag{4}$$

by finding an analytic expression involving the parameters q_1, q_2, q_3, $\vec{r}$, $\vec{r}_1$, $\vec{r}_2$ and $\vec{r}_3$. **When calculating the expression $\int \mathrm{d}^3 r' \, g(\vec{r} - \vec{r}\,') \, \rho(\vec{r}\,')$, carefully distiguish independent arguments of functions, integration variables, and parameters!!!**

(b.) Assume $q_1 = q_2 = q_3 = 0.04\,\mathrm{C}$, $\vec{r}_1 = \vec{0}$, $\vec{r}_2 = (3.5\,\mathrm{m})\,\hat{\mathrm{e}}_x$, and $\vec{r}_3 = (1.5\,\mathrm{m})\,\hat{\mathrm{e}}_y$. Calculate the quantities

$$\Phi(\vec{r}_a) = \,?\,, \qquad \vec{E}(\vec{r}_a) = \,?\,, \qquad \vec{r}_a = (1.7\,\mathrm{m})\,\hat{\mathrm{e}}_x + (1.5\,\mathrm{m})\,\hat{\mathrm{e}}_y + (0.1\,\mathrm{m})\,\hat{\mathrm{e}}_z \,, \tag{5}$$

and

$$\Phi(\vec{r}_b) = \,?\,, \qquad \vec{E}(\vec{r}_b) = \,?\,, \qquad \vec{r}_b = (-1.1\,\mathrm{m})\,\hat{\mathrm{e}}_x + (-1.5\,\mathrm{m})\,\hat{\mathrm{e}}_y + (0.7\,\mathrm{m})\,\hat{\mathrm{e}}_z \,. \tag{6}$$

Give numerical results for all three vector components of the electric fields at $\vec{r}_a$ and $\vec{r}_b$. Express your results in SI mksA units, preferably, to an accuracy of at least 4 decimals! The vacuum permittivity is $\epsilon_0 = 8.8542 \times 10^{-12}\mathrm{C\,V^{-1}\,m^{-1}}$, so that $1/(4\pi\epsilon_0) = 8.9875 \times 10^{9}\mathrm{N\,m^2/C^2}$.

(c.) Finally, calculate

$$\Phi_{\mathrm{diff}} = \Phi(\vec{r}_b) - \Phi(\vec{r}_a) \,. \tag{7}$$

Express your results in SI mksA units, preferably, to an accuracy of at least 4 decimals!

The tasks are due Tuesday, 16-APR-2026. Have fun doing the problems!