Task 1 (40 points) (Non–Uniform Convergence) Investigate the potential given in Eq. (6.75) of the lecture notes,

$$\Phi\left(x, y\right) = \frac{4\Phi_0}{\pi} \sum_{n=0}^{\infty} \frac{\sin\left[\left(2n+1\right)\pi x/a\right]}{2n+1} \, \frac{\sinh\left[\left(2n+1\right)\pi y/a\right]}{\sinh\left[\left(2n+1\right)\pi b/a\right]} \,. \tag{1}$$

Show the following properties which illustrate the non-uniform convergence at the point $x = 0$, and $y = b$,

$$\lim_{\epsilon \to 0^+} \Phi(0, b - \epsilon) = 0 \,, \qquad \lim_{\epsilon \to 0^+} \Phi(\epsilon, b) = \Phi_0 \,. \tag{2}$$

Task 2 (40 points) (Laplace Equation and Potential) Investigate the potential given in Eq. (6.90) of the lecture notes,

$$\Phi(x, y) = \frac{V_0}{\mathrm{i}\pi} \, \ln\left(\frac{\mathrm{e}^{\mathrm{i}\pi y/b} + \mathrm{e}^{\pi x/b}}{\mathrm{e}^{-\mathrm{i}\pi y/b} + \mathrm{e}^{\pi x/b}}\right) \,. \tag{3}$$

Show that, for real and positive $x > 0$, and $0 < y < b$, the result for $\Phi(x, y)$ is real rather than complex. Then, plot the potential given in Eq. (3), as a function of x and y, for your preferred choice of the parameter b, over a suitable range of x and y variables.

Task 3 (40 points) (Non–Uniform Convergence) Investigate, once more, Eq. (6.90) of the lecture notes, which is given above in Eq. (3). Show that

$$\lim_{\epsilon \to 0^+} \Phi(\epsilon, b) = 0 \,. \tag{4}$$

Furthermore, show that

$$\lim_{\epsilon \to 0^+} \Phi(0, b - \epsilon) = V_0 \,. \tag{5}$$

For the latter relation, use the result

$$\lim_{\eta \to 0^+} \ln(-1 \pm \mathrm{i}\eta) = \pm \mathrm{i}\pi \,. \tag{6}$$

Show that, as you calculate $\lim_{\epsilon \to 0^+} \Phi(0, b - \epsilon)$, you obtain an expression of the type given in Eq. (6), where the imaginary part of the argument of the logarithm is positive.

The tasks are due Thursday, 08–MAY–2025.