Task 1 (80 points) (Laplace Equation) You are given a two-dimensional wedge-shaped region between $\theta = 0$ and $\theta = 60^\circ$, which extends in the radial distance from $\rho = |\vec{\rho}| = \rho_a$ to $\rho = |\vec{\rho}| = \rho_b$. This constitutes a "piece of cake" with a "bite taken from the inside". Furthermore, you are given the boundary conditions

$$\Phi(|\vec{\rho}| = a, \theta) = f_a(\theta) = \Phi_a = \Phi_0\,, \qquad \Phi(|\vec{\rho}| = b, \theta) = f_b(\theta) = \Phi_b = \Phi_0\,, \tag{1}$$

i.e., a constant potential on the inner and outer edge of the "piece of cake", as well as

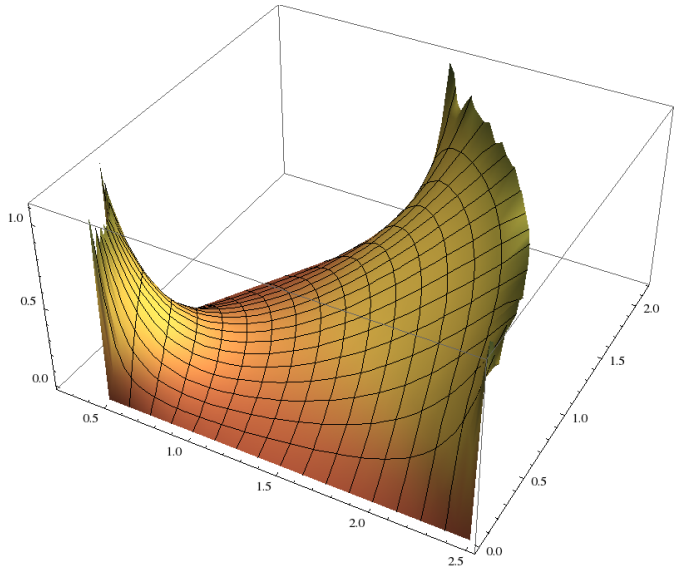
$$\Phi(\rho, \theta = 0) = \Phi(\rho, \theta = \beta) = 0\,. \tag{2}$$

The latter condition implies a vanishing potential on the left and right stripes of the "piece of the cake". Solve the Laplace boundary-value problem

$$\vec{\nabla}^2 \Phi(\vec{\rho}) = 0\,, \tag{3}$$

using a suitable series expansion of the potential. Consult the lecture notes.

Plot the solution. For the parameters $\rho_a = 0.5$, $\rho_b = 2.5$, and $\Phi_0 = 0.95$, show that one obtains (using a suitable number of expansion coefficients) a plot similar to



The tasks are due Thursday, 08–MAY–2025.