Task 1 (30 points)

Consider the retarded Green function of the harmonic oscillator,

$$\ddot{g_R}\,(t-t') \;+\gamma\,\dot{g_R}\,(t-t') + \omega_0^2\, g_R\,(t-t') = \delta\,(t-t')\ . \tag{1}$$

By first transforming to Fourier space and then backtransforming to the the time domain, show that

$$g(t-t') = 2\,\Theta(t-t')\,\mathrm{e}^{-\gamma(t-t')/2}\,\frac{\sin\left(\frac{1}{2}\,\sqrt{4\omega_0^2-\gamma^2}\,(t-t')\right)}{\sqrt{4\omega_0^2-\gamma^2}}\,. \tag{2}$$

Show all intermediate steps and show the integration contours; be explicit. Write a short essay on the questions: Why is this Green function naturally obtained as the retarded Green function? Why do you have to change the sign of the damping term in order to obtain the advanced Green function? Consider the effect of time reversal on the equation of motion, and provide an illustrative discussion.

Task 2 (80 points)

With Green function techniques (see task 1), based on the equation $x(t) = \int_{-\infty}^{\infty} \mathrm{d}t'\, g_R(t-t')\, f(t')$, investigate the damped harmonic oscillator. The differential equation is

$$\ddot{x} + \gamma\,\dot{x} + \omega_0^2\, x = f(t)\,. \tag{3}$$

Solve the differential equation (3) with the help of Green function techniques, where

$$f(t') = v_0\,\delta(t') \tag{4}$$

is an instantaneous "force push" at time $t = 0$. Use Green function techniques to obtain the solution under the boundary conditions $x(-\infty) = 0$, and $\dot{x}(-\infty) = 0$. Interpret the constant v_0 in terms of the velocity change before and after the "force push", and verify your result by explicit differentiation of your obtained solution near $t = 0$ (e.g., for infinitesimally positive and negative t).

Task 3 (80 points)

Same as task 2, but for the force term

$$f(t') = f_0\,\Theta(-t')\,\exp(t'/\tau)\,, \tag{5}$$

again with vanishing boundary conditions in the infinite past, $x(-\infty) = 0$, and $\dot{x}(-\infty) = 0$. Calculate $x(t) = \int_{-\infty}^{\infty} \mathrm{d}t'\, \ddot{g_R}(t-t')\, f(t')$, distinguishing the cases $t > 0$ and $t < 0$, plot the resulting $x(t)$ for numerical parameters of your choice, and interpret your result physically. In particular, show that the solution $x(t)$ and its derivative are continuous at $t = 0$. You may utilize the hints given during the lecture, but please formulate your thoughts so that it becomes clear that you show all your work.

Task 4 (50 extra points)

In the lecture, we had talked about the Lorentz transformation being applied to the Maxwell equations. Try to look up relevant formulas from the literature, and explain why magnetic fields can transform into electric fields under Lorentz transformations, and vice versa. Outline how you would calculate the transformed fields under Lorentz boosts. Explain why the electric and magnetic fields do not simply transform as four-vectors. This task is for the geeks and those who are otherwise underwhelmed by the difficulty of the course.

The tasks are due Tuesday, 11–MAR–2025.