Task 1 (30 points)
In the lecture, we had defined a rotation matrix $\mathbb{R}(\theta)$ and a projection matrix $\mathbb{P}_x$ as follows:

$$\mathbb{R}(\vartheta) = \begin{pmatrix} \cos\vartheta & -\sin\vartheta \\ \sin\vartheta & \cos\vartheta \end{pmatrix}, \qquad \mathbb{P}_x = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{1}$$

Calculate the commutator $\mathbb{C} = [\mathbb{R}(\vartheta), \mathbb{P}_x]$ and interpret your result geometrically! In particular, calculate the eigenvalues and eigenvectors of $\mathbb{C}$!

Task 2 (30 points)
In the lecture, we had defined a matrix representation of a complex number z as $\mathbb{M}(z)$. Show that the matrix representation of the reciprocal of a complex number, $\mathbb{M}(z^{-1})$, is equal to the inverse of the matrix representation of z, $[\mathbb{M}(z)]^{-1}$, by calculating both sides of the equation "from first principle",

$$\mathbb{M}(z^{-1}) = [\mathbb{M}(z)]^{-1} \,. \tag{2}$$

Show your work!

Task 3 (30 points)
Write a (very) short essay on the question (there might be a certain overlap of this question with the extra exercise set last time): How would you evaluate a line integral

$$I = \int_P \vec{F}(\vec{r}) \cdot \mathrm{d}\vec{r} \tag{3}$$

where P is a nontrivial path (say, with curves) and $\vec{F}(\vec{r})$ is a vector-valued function of a vector-valued variable $\vec{r}$ (the position). Hint: Start from a parameterisation $\vec{r} = \vec{r}_P(t)$, where t is the time and $\vec{r}_P(t)$ is the position of the object on the path P at time t.

Now, please delineate in detail: How would your task become easier if

$$\vec{F}(\vec{r}) = -\vec{\nabla} f(\vec{r}) \,, \tag{4}$$

where $f(\vec{r})$ is a scalar function (a potential). (Please note: The minus sign is inserted only for convention and has no special meaning.)

The tasks are due Thursday, 06-FEB-2025. Have fun doing the problems!