Syllabus for PHYSICS 6111 / Graduate Electrodynamics I

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Overview

We will discuss electrodynamics on the graduate level, with a special emphasis on mathematical methods universally applicable in both theoretical and experimental physics. On the physics side, we shall concentrate on static electric fields, and the solution of the Laplace and Poisson equations. On the mathematical side, we shall engage in a recap of basic mathematical techniques useful for physics, starting with complex numbers, and reaching over into special functions (notably, Bessel and hypergeometric functions). Also, a general knowledge of computer algebra systems will be a cornerstone of the course. Special emphasis will be laid on *Mathematica* and *Julia*, and the use of these systems for both theorists as well as experimentalists. The general "mantra" of the course is to learn universally applicable mathematical techniques, on examples derived from electrostatics. The learning objectives can be summarized as follows: Recap of complex numbers and special functions, with an emphasis on applications in physics. Recap of Taylor expansions and their geometric interpretation. Computer algebra systems and applications in electrostatics. Hilbert space, integrals, series expansions and applications in electrostatics. Multipole decompositions. Further topics. Projects.

Contents

1. **Complex Numbers and Calculus (Optional Recap).** Elementary Operations. Complex Numbers and 2×2 Matrices. Complex Square Root and Branch Cuts.

2. **Cauchy's Residue Theorem and Branch Cuts (Optional Recap).** Taylor Expansions. Laurent Series. Mittag–Leffler Expansions. Dispersion Relations.

3. **Differential Equations and Green Functions (Optional Recap).** Different Perspectives on Green Functions. Solution to Boundary–Value Problems. Green Functions and Poisson Equation.

4. **Maxwell Equations and Electrostatics.** Basic Properties of the Maxwell Equations. Maxwell Equations in Vacuum. Maxwell Equations and Electromagnetic Unit Systems. Electrostatic Energy Density and Self–Energy.

5. **Poisson Equation and Multipole Decompositions.** Spherical Harmonics and Their Visualization. Coordinate Systems and Special Functions. Spherical Harmonics. Multipole Expansion of Charge Distributions. Multipole Expansion in an External Field.

6. **Laplace Equation and Series Expansions.** Coordinate Systems and Special Functions. Laplace Equation in Different Geometries. Boundary Conditions.

7. **Laplace Equation and Variational Calculus.** Multivariate Functions and Functional Derivatives. Second Functional Derivative and Minimum of a Functional. Variations with Constraints. Approximation using a Functional.

8. **Challenging Additions for Geeks (Optional Project).** Nontrivial Charge Distributions and Nontrivial Boundary Conditions. Laplace, Poisson Equations, and Boundary Conditions. Green Functions for Dirichlet and Neumann Problems. Dirichlet and Neumann Problems: Applications. Green Function, Multipoles and Nontrivial Boundary Conditions. Sources and Fields in a Spherical Shell.

The precise outline of each chapter may be adjusted dynamically during the semester, depending on the preparations and ambitions, and other boundary conditions. We will strive to include numerical calculations using the computer algebra systems Julia and Mathematica, and perhaps python.

Advice and Encouragement

Commensurate with the requirements of a graduate course, students are encouraged to supplement the material taught in the lecture by their own reading. Some guidance is given in the lectures, and questions are always welcome, but the main responsibility for the filling of gaps in background knowledge remains with the student. The course compiles material from various textbooks, notably [U. D. Jentschura, Advanced Classical Electrodynamics, World Scientific, Singapore (2017)]. Further reading on the mathematical aspects of the course includes [R. Courant and D. Hilbert, Methods of Mathematical Physics—Volumes I and II, Interscience Publishers, New York (1966)], and [W. Magnus, F. Oberhettinger and R. P. Soni, Formulas and Theorems for the Special Functions of Mathematical Physics, Springer, New York (1966)], and [H. Bateman, Higher Transcendental Functions, Volumes I, II and III, McGraw–Hill, New York (1953)]. Lecture notes will be distributed.

Graded Exercises

The grading schedule of the course is as follows: There are graded exercises every week. These count from 60 to 150 points, typically. Furthermore, there may be one or two so-called "directed exercises" where you work on a specific problem in class, and then you are supposed to finish the work at home and hand in the exercise during the next lecture. The directed exercises (100 to 2000 points each) may or may not be announced. **The most important homework which is always due but never explicitly announced is reading the lecture notes, and, distributed notes. Actually doing this enables you to better perform in a hypothetical unannounced directed exercise as well as in an unannounced oral quiz near the start of a lecture, where we verify that basic wisdom has been learned from the distributed notes.** The points from the graded weekly exercises, from the directed exercises and from the oral quizzes are added near the end of the semester, to give a joint exercise grade. The exercise percentage grade counts 60% of the final grade.

Exercises will be available from **http://qedtheory.org/resources**.

Graded Exams

Two written exams will take place during the semester, and a final. The exams carry 150 to 200 points each and will be written during normal course hours. The percentage earned in the written exams counts 40% of the final grade. The final may replace the weakest exam, i.e., the exam percentage is calculated from the most favorable two exams out of the three: first exam, second exam, and final.

Final Exam

The final grading schedule follows the usual pattern. After weighted adding of the exercise and the exam grade (60% to 40%), an overall final grade is determined. From this final grade, ≥90% gives an A, ≥80% gives a B, ≥70% gives a C.

Make-up Policy

There are no make-ups for homework assignments. Students who anticipate being away for a class for a legitimate reason, should inform the instructor by e-mail ahead of class and give the reason for absence.

Appeals

If you believe an exception to a course rule should be made, you may file a written appeal. Appeals must be filed within one week of the occurrence of the circumstance that causes your appeal. Minor illness, lack of preparation, "I did poorly on two exams," non-emergency family events, oversleeping, "I forgot about it," etc., are not reasons for filing an appeal.

Unresolved Complaints about the Course

It is hoped that any complaints about the course can be resolved in a collegial manner through discussions with the instructor. However, if there are any complaints that cannot be resolved, you may take them up to Dr. Thomas Vojta, Physics Department Chairman.

Accessibility and Accommodations

It is the university's goal that learning experiences be as accessible as possible. If you anticipate or experience physical or academic barriers based on disability, please contact Student Disability Services at (573) 341-6655, dss@mst.edu, visit http://dss.mst.edu/ for information and to initiate the accommodation process.

Academic Dishonesty

Academic dishonesty, including cheating, plagiarism or sabotage, will be dealt with severely, and disruptive talking and other distractions will not be tolerated. See Student Academic Regulations at http://registrar.mst.edu/academicregs.

The use of material from prior instances of the course is not allowed and any indication of inappropriate use of such material will be dealt with harshly.

Furthermore, the use of artificial intelligence (AI) web interfaces and other AI-based aide in the course is not allowed.

Unless otherwise stated, only handwritten solutions will be accepted for the exercises, provided on physical paper (not electronically). Strong preference will be given for solutions written with a pen, not a pencil.

Title IX

The title IX policies, resources and reporting options are available online at http://titleix.mst.edu.