Task 1 (60 points) Please do the task absolutely independently!
We investigate complex numbers by way of example. Calculate the following quantities z_1, z_2, z_3, $|z_1|$, $|z_2|$, $|z_3|$, and θ_1, θ_2, and θ_3,

$$z_1 = (4.1 + \mathrm{i}\,5.3)^2 = |z_1| \, \exp(\mathrm{i}\theta_1)\,, \tag{1}$$

$$z_2 = \sqrt{4.0 + \mathrm{i}\,5.3} = |z_2| \, \exp(\mathrm{i}\theta_2)\,, \tag{2}$$

$$z_3 = (2.7 + \mathrm{i}\,7.3)^{1/2} = |z_3| \, \exp(\mathrm{i}\theta_3)\,. \tag{3}$$

In the case of multivalued functions, give the result on the principal branch of the Riemann sheet.

Task 2 (60 point) Please do the task absolutely independently!
We investigate complex numbers with the help of a computer algebra system, such as `julia` or `python`. In contrast to exercise #0, this time, with the help of a computer algebra system, not with the help of a calculator, calculate the following quantities z_1, z_2, z_3, $\mathrm{Re}(z_1)$, $\mathrm{Re}(z_2)$, $\mathrm{Re}(z_3)$, $\mathrm{Im}(z_1)$, $\mathrm{Im}(z_2)$, $\mathrm{Im}(z_3)$, $|z_1|$, $|z_2|$, $|z_3|$, and θ_1, θ_2, and θ_3,

$$z_1 = \sin(4.1 + \mathrm{i}\,5.3) = \mathrm{Re}(z_1) + \mathrm{i}\,\mathrm{Im}(z_1) = |z_1| \, \exp(\mathrm{i}\theta_1)\,, \tag{4}$$

$$z_2 = \cos(4.0 + \mathrm{i}\,5.3) = \mathrm{Re}(z_2) + \mathrm{i}\,\mathrm{Im}(z_2) = |z_2| \, \exp(\mathrm{i}\theta_2)\,, \tag{5}$$

$$z_3 = (2.7 + \mathrm{i}\,7.3)^{1/2} = \mathrm{Re}(z_3) + \mathrm{i}\,\mathrm{Im}(z_3) = |z_3| \, \exp(\mathrm{i}\theta_3)\,. \tag{6}$$

In the case of the square root, give the result on the principal branch of the Riemann sheet, i.e., please give the “usual value” of the square root. Details will be discussed in the lecture. You may use computer algebra, but you must include a complete program listing. Specifically, Use the functions `numpy.arctan2` in `python` or `atan` in `julia`. Submit a transcript of your computer algebra session (“program code”).

Task 3 (60 points) Please do the task absolutely independently!
You are given the (3×3)-matrix

$$\mathbb{M} = \begin{pmatrix} 0 & \mathrm{i} & 0 \\ -\mathrm{i} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{7}$$

Determine the eigenvalues and eigenvectors of $\mathbb{M}$.

Task 4 (40 points) Please do the task absolutely independently!
The general statement is that, if you matrix-multiply an $m \times n$ matrix with an $n \times p$ matrix, you get an $m \times p$ matrix. Illustrate this statenment graphically (by writing a sample matrix) for the case $m = 4$, $n = 3$, and $p = 2$. Discuss how your graphical representation, with row and column vectors clearly identified, is compatible with the formula

$$\mathbb{C} = \mathbb{A} \cdot \mathbb{B}\,, \qquad C_{ik} = \sum_{j=1}^{n} A_{ij}\, B_{jk}\,. \tag{8}$$

What would be the range for the indices i and k in the example above?

Task 5 (40 points) Please do the task absolutely independently!
Evaluate the product

$$(4.3 + \mathrm{i}\,5.2) \times (2.3 + \mathrm{i}\,7.3) = \mathrm{Re}(z) + \mathrm{i}\,\mathrm{Im}(z) \tag{9}$$

(i) by direct complex multiplication, and *(ii)* by first determining the (2×2) matrices corresponding to the two complex numbers, as introduced in the lecture, then performing the matrix multiplication, and then, identifying the complex number that the resulting product matrix corresponds to. I.e., evaluate $\mathbb{M}(z) = \mathbb{M}(4.3 + \mathrm{i}\,5.2) \cdot \mathbb{M}(2.3 + \mathrm{i}\,7.3)$ and identify z.

The most important homework, never announced but always due, is to read and understand the lecture notes, and the material covered in the lecture, so that questions on them can be answered at the beginning of each session. The tasks are due on Thursday, 30–JAN–2025. No extension whatsoever will be given.