
Task 1 (100 points)
Consider once more the expansion

$$g(\vec{r}, \vec{r}') = -\frac{1}{4\pi} \sum_{m=-\infty}^{\infty} \mathrm{e}^{\mathrm{i}m\left(\varphi - \varphi'\right)} \int_0^\infty \mathrm{d}k \, \mathrm{e}^{-k(z_> - z_<)} \, J_m\left(k\rho\right) \, J_m\left(k\rho'\right) \tag{1}$$

derived in the lecture, for the case

$$\vec{r} = \vec{r}_1 = 1.5\hat{\mathrm{e}}_x + 2.3\hat{\mathrm{e}}_y + 0.8\hat{\mathrm{e}}_z \,, \qquad \vec{r}' = \vec{r}_2 = 0.3\hat{\mathrm{e}}_x + 0.4\hat{\mathrm{e}}_y + 1.1\hat{\mathrm{e}}_z \,. \tag{2}$$

Convert $\vec{r}_1$ and $\vec{r}_1$ into cylindrical coordinates ρ_1, φ_1, z_1 and ρ_2, φ_2, and z_2. Then, evaluate the numerical sum

$$g(\vec{r}, \vec{r}') \approx \sum_{m=-10}^{10} \mathrm{e}^{\mathrm{i}m(\varphi_1 - \varphi_2)} \int_0^\infty \mathrm{d}k \, \mathrm{e}^{-k(z_2 - z_1)} \, J_m\left(k\rho_1\right) \, J_m\left(k\rho_2\right) \tag{3}$$

with the help of your favorite computer system. Show that the exact result (which one?) is reproduced to better than six decimals.

The tasks are due Thursday, 21–NOV–2024. Have fun doing the problems!