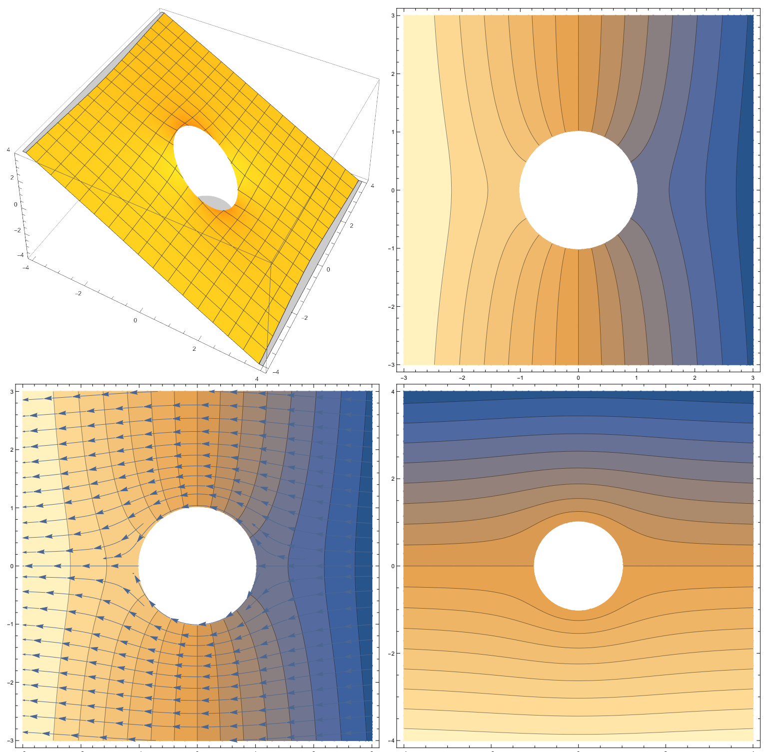
Physics/6403 **Mathematical Physics I** **Dr. Ulrich Jentschura** **Fall Semester 2024**
Missouri S & T *[Project Exercise 1 (Voluntary)]* **Tuesday, 08–OCT–2024**

Task 1 (unmarked). Do an extended recap of all material covered in the lecture so far and contemplate, repeatedly, all points which you have not fully digested, until you feel that you master the subject in depth.

Task 2 (points to be allocated). Consider once more the potential flow for a cylinder as given in the following various visualizations illustrating the velocity potential around a cylinder. Top row: Velocity potential (3D plot, left panel, and contour plot, right panel). Bottom row: Equipotential lines of the velocity potential and field lines of the velocity field (left panel) and equipotential lines of the stream function (right panel).



(a) Write a code (in your favorite language, python, julia, MatLab, *or whatever*), which reproduces the plots for typical parameters (cylinder radius a, oncoming velocity u_∞) of your choice.

(b) In exercise #5, we had derived various asymptotic forms for the lines of constant stream function, in the regime of large x. Look these up. These asymptotic forms should match the numerical solutions pretty well, and they should conincide, for large x, with the apparent asymptotic shape of the contours in the right bottom panel above. Write a code which calculates the stream lines numerically and convince yourself that the asymptotic relation is fulfilled.

Task 3 (points to be allocated). Investigate the complex potential

$$\overline{w} = -\mathrm{i}\frac{\Gamma}{2\pi} \ln\left(\frac{z^*}{a}\right) , \tag{1}$$

where $z = x + \mathrm{i}y$ and $z^* = x - \mathrm{i}y$. (Watch the complex conjugate!) Convince yourself that the velocity potential $\phi = \mathrm{Re}\,(\overline{w})$ flips sign in comparison to $w = -\mathrm{i}\frac{\Gamma}{2\pi} \ln\left(\frac{z}{a}\right)$, which could otherwise indicate that $\overline{w}$ might be a suitable complex potential for clockwise flow. However, the Cauchy–Riemann equations are probably not fulfilled. Investigate this problem, on the basis of explicit differentiations with respect to x and y.

The tasks are due on Thursday, 17–OCT–2024.