Setting. In the first part of the lecture, we had associated complex numbers with (active) two-dimensional rotations (and the modulus of a complex rotation with a "stretch factor" for the complex plane). Now, we deal with passive rotations. Consider rotations in two-dimensional space. (Unmarked part of the exercise: Convince yourself that an active rotation by $+\theta$ is equivalent to a passive rotation by $-\theta$, as far as the coordinates are concerned.)

Task 1 (40 points). Interpret the "usual" basis vectors as the covariant ones. The Cartesian basis vectors are written as $\vec{e}_i = \hat{\mathrm{e}}_i$ with $i = 1, 2$. Furthermore, interpret the $x^i = (x, y)$ with $i = 1, 2$ as the "usual" "contravariant" Cartesian coordinates.

Write the basis vectors of the rotated basis (angle θ, passive rotation), denoted as $\widetilde{\vec{e}}_i$, as a function of the Cartesian basis vectors $\vec{e}_i = \hat{\mathrm{e}}_i$. Determine the (2×2)-matrix of the $\partial x^j / \mathrm{d}\tilde{x}^i$ which enters the relation

$$\widetilde{\vec{e}}_i = \sum_{j=1}^{2} \frac{\partial x^j}{\mathrm{d}\widetilde{x}^i} \, \vec{e}_j \,, \tag{1}$$

where $\vec{e}_j$ is the jth usual Cartesian basis vector.

Task 2 (40 points). Based on the invariance of the physical vector $\vec{V}$ under a passive rotation,

$$\widetilde{\vec{V}} = \sum_{i=1}^{2} \widetilde{x}^i \, \widetilde{\vec{e}}_i = \sum_{i=1}^{2} x^i \, \vec{e}_i = \vec{V} \,, \tag{2}$$

write the $\widetilde{x}^i$ as a function of the x^i and determine the matrix which mediates the transformation of the infinitesimal coordinate changes $\mathrm{d}\widetilde{x}^i$ and $\mathrm{d}x^i$.

The tasks are easy but require you to think about the definitions carefully. SIGNS MATTER IN YOUR SOLUTION! The expression $\sin\theta$ is not equal to $-\sin\theta$.

The tasks are due on Tuesday, 29–OCT–2024.