Task 1 (30 points)
Recall some basic aspects of linear algebra. Show that, under suitable conditions which you do not need to discuss, the eigenvectors $\vec{u}_i$ with $i = 1, \ldots, n$ of an $(n \times n)$–Hermitian matrix $\mathbb{M}$ (look up the definition if necessary) form a complete orthogonal set of eigenvectors. How do you convert the ortho*gonal to an ortho*normal set? Express the matrix $\mathbb{U}$ which diagonalizes $\mathbb{M}$,

$$\mathbb{D} = \mathbb{U}^{-1} \cdot \mathbb{M} \cdot \mathbb{U} \tag{1}$$

in terms of the vectors $\vec{u}_i$. How are $\mathbb{U}^\dagger$ and $\mathbb{U}$ related? (The matrix $\mathbb{D}$ is diagonal.)

This task should not take longer than 20 minutes. Otherwise, please use the opportunity to look up notes on linear algebra.

Task 2 (30 points)
You were given the definition of the Pauli matrices σ_x, σ_y and σ_z. If necessary, look up their definitions. Find their eigenvalues (hint: maybe -1, 1) and normalized (to unity) eigenvectors and show that the latter form an orthonormal set.

This task should not take longer than 20 minutes. Otherwise, please use the opportunity to look up notes on linear algebra.

The tasks are due Thursday, Tuesday, 15–OCT–2024.