Task 1 (50 points)
Consider the parameters

$$\vec{r} = \vec{r}_1 = 5.4\,\hat{e}_x + 3.4\hat{e}_y + 2.3\hat{e}_z\,, \qquad \vec{r}' = \vec{r}_2 = 5.1\,\hat{e}_x + 3.3\hat{e}_y + 2.2\hat{e}_z\,. \tag{1}$$

Define the terms

$$\begin{aligned} T_\ell &= \sum_{m=-\ell}^{\ell} \frac{4\pi}{2\ell+1}\,\frac{r_<^\ell}{r_>^{\ell+1}}\,Y_{\ell m}(\theta,\varphi)\,Y^*_{\ell m}(\theta',\varphi') \\ &= \sum_{m=-\ell}^{\ell} \frac{4\pi}{2\ell+1}\,\frac{r_2^\ell}{r_1^{\ell+1}}\,Y_{\ell m}(\theta_1,\varphi_1)\,Y^*_{\ell m}(\theta_2,\varphi_2)\,, \end{aligned} \tag{2}$$

where the second line is just a trivial specialization of the first, to the case $\vec{r} = \vec{r}_1$ and $\vec{r}' = \vec{r}_2$, and we anticipate that $r_2 = r_<$, and $r_1 = r_>$ (why?). Write a <u>computer symbolic program</u> which calculates, explicitly and numerically,

$$T_\ell\,, \qquad 0 \le \ell \le 20\,. \tag{3}$$

Show that the sum converges **slowly**, and that the result

$$\sum_{\ell=0}^{20} T_\ell \qquad \text{approximates only about 75\% of the full result for} \qquad \frac{1}{|\vec{r}-\vec{r}'|}\,. \tag{4}$$

Then, calculate the first 201 terms $0 \le \ell \le 200$ and show that the full result for $1/|\vec{r}-\vec{r}'|$ is obtained to better than 90% agreement.

The tasks are due Thursday, 18–APR–2024, with a possible extension.