=== For the Geeks and Nerds! ===
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Task 1 (UNMARKED, HOLIDAYS!) (Boundary–Value Problem) Define $\rho = \sqrt{x^2 + y^2}$ as the distance from the z axis. For the Laplace equation $\vec{\nabla}^2 \Phi(\vec{r}) = 0$, you are given a boundary-value problem for a cylinder, whose symmetry axis coincides with the z axis, and which extends from $z = -L/2$ to $z = L/2$ (i.e., the cylinder has a length L). The radius of the cylinder is a. The walls of the cylinder are held at potential $\Phi = 0$ (grounded), and both end caps held at the electrostatic potential

$$\Phi(x, y, z = \pm\tfrac{1}{2}\,L) = \Phi_0 \left(1 - \frac{x^2 + y^2}{a^2}\right) = \Phi_0 \left(1 - \frac{\rho^2}{a^2}\right) \tag{1}$$

Explain why a suitable *ansatz* for the solution of this boundary-value problem is

$$\Phi(x, y, z) = \sum_{mn} c_{mn}\, J_m\left(\xi_{mn}\,\frac{\rho}{a}\right) \cosh\left(\xi_{mn}\,\frac{z}{a}\right) \exp(\mathrm{i} m\varphi)\,. \tag{2}$$

Here, ξ_{mn} is the nth zero of the mth Bessel function,

$$J_m(\xi_{mn}) = 0\,, \qquad m = 0, 1, 2, \dots\,, \qquad n = 1, 2, 3, \dots\,, \tag{3}$$

Convince yourself that the only contributing terms have $m = 0$, and, on the basis of the theory of Bessel functions, determine c_{mn} and simplify the result as much as possible. (Only terms with $m = 0$ will contribute.) If you feel ambitious, plot the resultant series (the first terms in the sum over n) for the parameters $L = a = 10\,\mathrm{cm}$ (plotting the axes in units of centimeters), and exhibit the convergence toward the boundary condition.

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The tasks are unmarked, but I will be happy to have a look at your solutions!