
Task 1 (40 points)
Consider the three-dimensional generalization of a Taylor expansion,

$$f(\vec{r}) = f(\vec{0}) + \vec{r}\cdot\vec{\nabla} f(\vec{r})\Big|_{\vec{r}=\vec{0}} + \sum_{i,j=1}^{3} \frac{1}{2!} r_i\, r_j \; \frac{\partial}{\partial r_i}\frac{\partial}{\partial r_j} f(\vec{r})\Big|_{\vec{r}=\vec{0}} + \dots ,$$

$$\vec{r}\cdot\vec{\nabla} f(\vec{r})\Big|_{\vec{r}=\vec{0}} = \sum_{i=1}^{3} r_i \; \frac{\partial}{\partial r_i} f(\vec{r})\Big|_{\vec{r}=\vec{0}} . \tag{1}$$

Consider the function

$$f(\vec{r}) = f(x,y,z) = \exp\left[-(x + x^2 + y^2 + 2z^2)\right] \tag{2}$$

at the point $x = 0.1$, $y = 0.1$, and $z = 0.05$. Evaluate the first two terms of the three-dimensional Tyalor expansion at the origin explicitly and show that this expansion leads to the approximation

$$f(0.1, 0.1, 0.05) = 0.882496\cdots \approx 1 - 0.1 - 0.02 = 0.88 , \tag{3}$$

where the terms 0.1 and 0.02 are obtained from the two subleading terms in the Taylor expansion.

Task 2 (30 points)
Consider the function

$$f(x,y) = x^2 + y^2 + 3xy . \tag{4}$$

Show that its two-dimensional Taylor expansion, taken to second order, is equal to the function that you started from.

Task 3 (30 extra points)
Repeat **Task 3** for the function

$$f(x,y,z) = x + y + z + xy + x^3 + x^2 y + xyz , \tag{5}$$

in three-dimensional space. Show that it is reproduced by its third-order Taylor expansion.

Task 4 (30 points)
Calculate

$$Q_1 = \vec{\nabla}^2 \exp(-r/a) , \qquad Q_2 = \vec{\nabla} \exp(-r/a) , \qquad Q_2 = \vec{\nabla} \times [\hat{e}_z \exp(-r/a)] , \tag{6}$$

where $r = \sqrt{x^2 + y^2 + z^2}$ and a is a constant of dimension length.

The tasks are due Tuesday, 22–FEB-2024. Have fun doing the problems!