

**Task 1** (60 points) **Please do the task absolutely independently!**

We investigate complex numbers by way of example. Calculate the following quantities  $z_1$ ,  $z_2$ ,  $z_3$ ,  $|z_1|$ ,  $|z_2|$ ,  $|z_3|$ , and  $\theta_1$ ,  $\theta_2$ , and  $\theta_3$ ,

$$z_1 = (4.3 + i5.3)^2 = |z_1| \exp(i\theta_1), \quad (1)$$

$$z_2 = \sqrt{4.3 + i5.3} = |z_2| \exp(i\theta_2), \quad (2)$$

$$z_3 = (2.3 + i7.3)^{1/2} = |z_3| \exp(i\theta_3). \quad (3)$$

In the case of multivalued functions, give the result on the principal branch of the Riemann sheet.

**Task 2** (60 point) **Please do the task absolutely independently!**

We investigate complex numbers **with the help of a computer algebra system, such as julia or python. In contrast to exercise #0, this time, with the help of a computer algebra system, not with the help of a calculator**, calculate the following quantities  $z_1$ ,  $z_2$ ,  $z_3$ ,  $\operatorname{Re}(z_1)$ ,  $\operatorname{Re}(z_2)$ ,  $\operatorname{Re}(z_3)$ ,  $\operatorname{Im}(z_1)$ ,  $\operatorname{Im}(z_2)$ ,  $\operatorname{Im}(z_3)$ ,  $|z_1|$ ,  $|z_2|$ ,  $|z_3|$ , and  $\theta_1$ ,  $\theta_2$ , and  $\theta_3$ ,

$$z_1 = \sin(4.3 + i5.3) = \operatorname{Re}(z_1) + i \operatorname{Im}(z_1) = |z_1| \exp(i\theta_1), \quad (4)$$

$$z_2 = \cos(4.3 + i5.3) = \operatorname{Re}(z_2) + i \operatorname{Im}(z_2) = |z_2| \exp(i\theta_2), \quad (5)$$

$$z_3 = (2.3 + i7.3)^{1/2} = \operatorname{Re}(z_3) + i \operatorname{Im}(z_3) = |z_3| \exp(i\theta_3). \quad (6)$$

In the case of the square root, give the result on the principal branch of the Riemann sheet, i.e., please give the “usual value” of the square root. Details will be discussed in the lecture. **You may use computer algebra, but you must include a complete program listing. Specifically, Use the functions `numpy.arctan2` in python or `atan` in julia. Submit a transcript of your computer algebra session (“program code”).**

**Task 3** (60 points) **Please do the task absolutely independently!**

You are given the  $(3 \times 3)$ -matrix

$$\mathbb{M} = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (7)$$

Determine the eigenvalues and eigenvectors of  $\mathbb{M}$ .

**Task 4** (40 points) **Please do the task absolutely independently!**

Evaluate the product

$$(4.3 + i5.2) \times (2.3 + i7.3) = \operatorname{Re}(z) + i \operatorname{Im}(z) \quad (8)$$

(i) by direct complex multiplication, and (ii) by first determining the  $(2 \times 2)$  matrices corresponding to the two complex numbers, as introduced in the lecture, then performing the matrix multiplication, and then, identifying the complex number that the resulting product matrix corresponds to. I.e., evaluate  $\mathbb{M}(z) = \mathbb{M}(4.3 + i5.2) \cdot \mathbb{M}(2.3 + i7.3)$  and identify  $z$ .

The most important homework, never announced but always due, is to read and understand the lecture notes, and the material covered in the lecture, so that questions on them can be answered at the beginning of each session. The tasks are due on Thursday, 25–JAN–2024. No extension whatsoever will be given.