Task 1 (60 points) Please do the task absolutely independently!
We investigate complex numbers with the help of a computer algebra system, such as `julia` or `python`. In contrast to exercise #0, this time, with the help of a computer algebra system, not with the help of a calculator, calculate the following quantities z_1, z_2, z_3, $|z_1|$, $|z_2|$, $|z_3|$, and θ_1, θ_2, and θ_3,

$$z_1 = (4.3 + \mathrm{i}\,5.3)^2 = |z_1| \exp(\mathrm{i}\theta_1)\,, \tag{1}$$

$$z_2 = \sqrt{4.3 + \mathrm{i}\,5.3} = |z_2| \exp(\mathrm{i}\theta_2)\,, \tag{2}$$

$$z_3 = (2.3 + \mathrm{i}\,7.3)^{1/2} = |z_3| \exp(\mathrm{i}\theta_3)\,. \tag{3}$$

In the case of multivalued functions, give the result on the principal branch of the Riemann sheet. For the latter task, define the concept of the branch cut of the complex square root and choose a branch cut. Use the functions `numpy.arctan2` in `python` or `atan` in `julia`. Submit a transcript of your computer algebra session ("program code").

Task 2 (60 points) Please do the task absolutely independently!
Calculate the following quantities z_4, z_5, $|z_4|$, $|z_5|$, and θ_4, θ_5, with the help of a computer algebra system,

$$z_4 = \exp(-4.3 + \mathrm{i}\,5.3) = |z_4| \exp(\mathrm{i}\theta_4)\,, \tag{4}$$

$$z_5 = \cos(4.3 + \mathrm{i}\,5.3) = |z_5| \exp(\mathrm{i}\theta_5)\,. \tag{5}$$

Then, write a "loop" which calculates

$$z_6 = \exp(\mathrm{i}(2.3x - 6.9 \times 10^8 t))\,,$$
$$\text{for} \quad x \in \{0.1, 0.2, \dots, 1.0\}\,, \quad t \in \{0.1 \times 10^{-7}, 0.2 \times 10^{-7}, \dots, 1.0 \times 10^{-7}\}\,. \tag{6}$$

Additional task for 10 extra points: How is the last task numerically related to electromagnetic waves?

Task 3 (60 points)
(a) Do a personal recap of the Maxwell equations. Show that the divergence of the Ampere–Maxwell law gives the time derivative of Gauss's law. Present your notes.

(b) Then, starting from the derive the differential as well as the charge conservation law, show how to obtain the integral form for the charge conservation law,

$$\vec{\nabla} \cdot \vec{J}(\vec{r}, t) = -\frac{\partial}{\partial t} \rho(\vec{r}, t) \quad \Leftrightarrow \quad \int_{\partial V} \vec{J}(\vec{r}, t) \cdot \mathrm{d}A = -\frac{\mathrm{d}}{\mathrm{d}t} \int_V \rho(\vec{r}, t) \mathrm{d}V\,, \tag{7}$$

and vice versa (both directions!). Present a complete derivation! (You may consult lecture notes for PHYSICS 6111.) (Symbols are defined as in the lecture (notes).)

(c) Which equation results for a time-independent charge density? Intuitively speaking, what kind of a (nonvanishing) current distribution is compatible with a time-independent charge distribution? Draw a sketch!

Task 4 (20 points) Explain in your own words, but in complete English sentences, why the case $\epsilon_r \to \infty$ for the relative permittivity corresponds to a "perfectly polarizable medium" in the static limit.

Task 5 (40 points)
Show that

$$\vec{\nabla} \cdot \left[\vec{E}\,(\vec{r}, t) \times \vec{H}\,(\vec{r}, t) \right] = -\vec{E}\,(\vec{r}, t) \cdot \vec{\nabla} \times \vec{H}\,(\vec{r}, t) + \vec{H}\,(\vec{r}, t) \cdot \vec{\nabla} \times \vec{E}\,(\vec{r}, t) \tag{8}$$

by reference to the Levi-Civita tensor ϵ_{ijk}. Show how to get from the phenomenological formulation of the Poynting theorem (which involves $\vec{D}$ and $\vec{H}$ and is valid for the presence of a dielectric) to the fundamental formulation ("in vacuum") of the Poynting theorem (which involves only $\vec{E}$ and $\vec{B}$).

The most important homework, never announced but always due, is to read and understand the lecture notes, and the material covered in the lecture, so that questions on them can be answered at the beginning of each session. The tasks are due on Tuesday, 05–SEP–2023 with a possible extension to Thursday, 07–SEP–2023 (but no additional extension whatsoever).