Task 1 (50 points)
Calculate the integral

$$\int_0^\infty \mathrm{d}x \, \frac{\sin(x)}{x} \tag{1}$$

by residue calculus. (Any other way of calculation will result in zero points!) Hint: Symmetrize the integrand, write the sin function in terms of exponentials, displace the integration slightly above or below the real axis, and close the integration contour along a circle at infinity.

Task 2 (30 points)
Calculate the Green function of the Poisson equation in three dimensions,

$$g(\vec{r} - \vec{r}') = -\frac{1}{4\pi|\vec{r} - \vec{r}'|} \tag{2}$$

by Fourier transforming to wave vector space, and backtransforming to position space.

Task 3 (30 points)
Show that $g(x - x')$ is a Green function of the one-dimensional Poisson equation,

$$g(x - x') = \frac{|x - x'|}{2}\,, \qquad \frac{\partial^2}{\partial x^2} g(x - x') = \delta(x - x')\,, \tag{3}$$

and that

$$\tilde{g}(x - x') = \Theta(x - x')\,(x - x') \tag{4}$$

also is a valid Green function of the one-dimensional Poisson equation,

$$\frac{\partial^2}{\partial x^2} \tilde{g}(x - x') = \delta(x - x')\,. \tag{5}$$

Also, show that

$$f(x - x') = g(x - x') - \tilde{g}(x - x') \tag{6}$$

is a solution of the homogeneous equation.

The tasks are due Tuesday, 14-FEB-2023. Have fun doing the problems!