Physics/6111 Electrodynamics I Dr. Ulrich Jentschura Spring Semester 2023

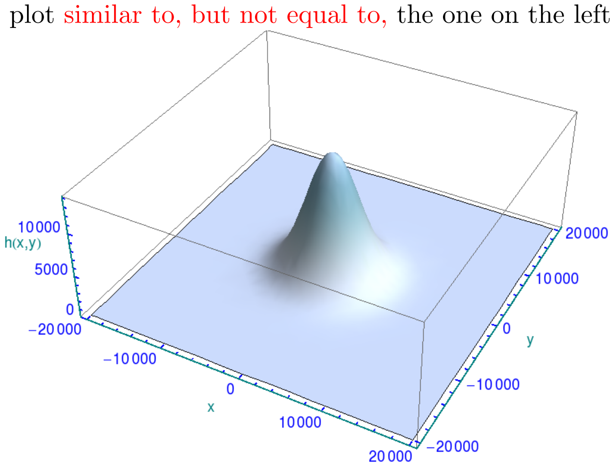
Missouri S & T (Rolla, Missouri) Exercise Sheet 2X 24–JAN–2023

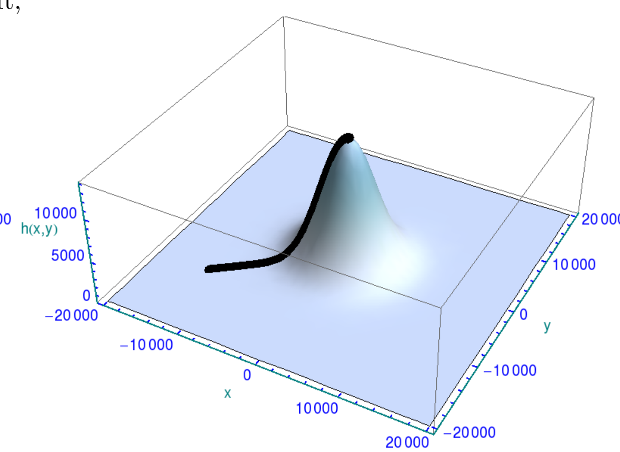
PLEASE READ THE TASKS CAREFULLY!!!

Task 1 (100 points)
Consider a mountain surface given by the equation

$$h(x,y) = A\,\exp[-B\,(x^2+y^2)]\,, \qquad A = 14200\,\mathrm{ft}\,, \qquad B = \frac{1}{2.5\times 10^7}\,\frac{1}{(\mathrm{ft}^2)}\,. \tag{1}$$

Here, ft is the unit of foot. Plot $h(x,y)$ over a meaningful range of x and y values. You might obtain a plot similar to, but not equal to, the one on the left,





but you should label the x and y, and z axes with appropriate physical units (which can be given in feet or meters). Then, consider the path

$$\vec{s}(t) = \hat{\mathrm{e}}_x\,s_x(t) + \hat{\mathrm{e}}_y\,s_y(t)\,, \qquad s_x(t) = s_{0x} + v_x\,t\,, \qquad s_y(t) = s_{0y} + v_y\,t\,,$$
$$s_{0x} = s_x(t=0) = -13500\,\mathrm{ft}\,, \quad s_{0y} = s_y(t=0) = -10000\,\mathrm{ft}\,, \quad v_x = \frac{13.5}{16}\,\frac{\mathrm{ft}}{\mathrm{s}}\,, \quad v_y = \frac{5}{8}\,\frac{\mathrm{ft}}{\mathrm{s}}\,, \tag{2}$$

where s stands for the unit of second. Plot $h(\vec{s}(t))$ over a the range $t \in (0, 16000\,\mathrm{s})$ and overlay the plot with the one obtained above. You might obtain a plot like the one on the right, above, but you should label the x and y, and z axes with appropriate physical units (which can be given in feet or meters).

Show, by an explicit and complete analytic evaluation in terms of the general parameters A, B, s_{0x} and s_{0y}, that

$$\frac{\mathrm{d}h(\vec{s}(t))}{\mathrm{d}t} = \vec{\nabla}h(\vec{r})\Big|_{\vec{r}=\vec{s}(t)} \cdot \frac{\mathrm{d}\vec{s}(t)}{\mathrm{d}t}\,. \tag{3}$$

Show every step and every intermediate result for every single quantity on the left and right-hand sides of the equation! Calculate the integral (numerically!)

$$\int_{t=0}^{t=16000\,\mathrm{s}} \mathrm{d}t\,\frac{\mathrm{d}h(\vec{s}(t))}{\mathrm{d}t} \approx 14200\,\mathrm{ft} \tag{4}$$

and interpret your result geometrically, looking at the plot. How fast do you reach the summit for the parameters $v_x = 6.75\,\mathrm{ft/s}$ and $v_y = 5\,\mathrm{ft/s}$, for an identical start point of your way up the mountain?

The tasks are due Tuesday, 31-JAN-2023. Have fun doing the problems!