Syllabus for PHYSICS 6403 / Mathematical Physics I

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Overview

We will discuss advanced mathematical methods absolutely indispensable for any research project in either experimental or theoretical physics. We start with complex variable theory and complex contour integration, with an emphasis on the indispensable practical knowledge regarding the application of these concepts "for serious physics research". We continue with a discussion of coordinate transformations, based of course on matrix representations of the coordinate transformations, and basic vector analysis. Topics will include, among others things, Stokes's theorem in both differential as well as integral form, and transformations into curvilinear coordinates, as well as Christoffel symbols and the different forms of gradient and divergence operators, in different coordinate systems (e.g., spherical and cylindrical). Tensors will be discussed. The separation ansatz for the solution of partial differential equations will be discussed and illustrated. A discussion of the most indispensable special functions necessary for physics research follows: orthogonal functions and solutions to ordinary differential equations, Gamma function, hypergeometric, confluent hypergeometric, Legendre, Laguerre, and Bessel functions, and Hermite polynomials. The course may end with a discussion of Green functions in one dimension, or, interactively, with discussions on any topics where students feel the need for a refreshment of their mathematical background knowledge. The necessity of diligence, and the presence of pitfalls in the mathematical discussions, will be highlighted.

Contents

1 Motivation

Why is mathematics so important in the description of nature?

2 Matrix Multiplication and Complex Arithmetic

(Hopefully) familiar results in a new light

3 Complex Numbers and Vector Fields

Two-dimensional vector fields and analogies with complex numbers

4 Basics of the Gradient Operator

Poisson equation, divergence and Stokes's theorem

5 Cauchy–Riemann Equations

Basics of complex functions of complex variables

6 Cauchy's Residue Theorem

A marvel of mathematics

7 Branch Cuts

Needed for physics

8 Conformality and Complex Potentials

Exploring the structure of complex functions

9 Toward the Kutta–Joukowsky Theorem

Mathematics and technology: not so far apart

10 Tensors: Fundamentals

Important and indispensable for physics

11 Tensors: Applications

Gradient, divergence, curl, Laplacian, in different coordinates

12 Differential Equations and Green Functions

Fundamental ingredients in the description of nature

Advice and Encouragement

Commensurate with the requirements of a graduate course, students are encouraged to supplement the material taught in the lecture by their own reading. Some guidance is given in the lectures, and questions are always welcome, but the main responsibility for the filling of gaps in background knowledge remains with the student. The course compiles material from various textbooks, found scattered in the literature. Some excerpts are taken from [R. Courant and D. Hilbert, Methods of Mathematical Physics—Volumes I and II, Interscience Publishers, New York (1966)], and [W. Magnus, F. Oberhettinger and R. P. Soni, Formulas and Theorems for the Special Functions of Mathematical Physics, Springer, New York (1966)], and [H. Bateman, Higher Transcendental Functions, Volumes I, II and III, McGraw–Hill, New York (1953)]. Lecture notes will be distributed.

Graded Exercises

The grading schedule of the course is as follows: There are graded exercises every week. These count from 60 to 150 points, typically. Furthermore, there may be one or two so-called "directed exercises" where you work on a specific problem in class, and then you are supposed to finish the work at home and hand in the exercise during the next lecture. The directed exercises (100 to 2000 points each) may or may not be announced. The most important homework which is always due but never explicitly announced is reading the lecture notes, and, distributed notes. Actually doing this enables you to better perform in a hypothetical unannounced directed exercise as well as in an unannounced oral quiz near the start of a lecture, where we verify that basic wisdom has been learned from the distributed notes. The points from the graded weekly exercises, from the directed exercises and from the oral quizzes are added near the end of the semester, to give a joint exercise grade. The exercise percentage grade counts 60% of the final grade.

Exercises will be available from **www.qedtheory.org/resources**.

Graded Exams

Two written exams will take place during the semester, and a final. The exams carry 150 to 200 points each and will be written during normal course hours. The percentage earned in the written exams counts 40% of the final grade. The final may replace the weakest exam, i.e., the exam percentage is calculated from the most favorable two exams out of the three: first exam, second exam, and final.

Final Exam

The final grading schedule follows the usual pattern. After weighted adding of the exercise and the exam grade (60% to 40%), an overall final grade is determined. From this final grade, $\geq$90% gives an A, $\geq$80% gives a B, $\geq$70% gives a C.

Make-up Policy

There are no make-ups for homework assignments. Students who anticipate being away for a class for a legitimate reason, should inform the instructor by e-mail ahead of class and give the reason for absence.

Appeals

If you believe an exception to a course rule should be made, you may file a written appeal. Appeals must be filed within one week of the occurrence of the circumstance that causes your appeal. Minor illness, lack of preparation, “I did poorly on two exams,” non-emergency family events, oversleeping, “I forgot about it,” etc., are not reasons for filing an appeal.

Unresolved Complaints about the Course

It is hoped that any complaints about the course can be resolved in a collegial manner through discussions with the instructor. However, if there are any complaints that cannot be resolved, you may take them up to Dr. Thomas Vojta, Physics Department Chairman.

Accessibility and Accommodations

It is the university’s goal that learning experiences be as accessible as possible. If you anticipate or experience physical or academic barriers based on disability, please contact Student Disability Services at (573) 341-6655, dss@mst.edu, visit http://dss.mst.edu/ for information and to initiate the accommodation process.

Academic Dishonesty

Academic dishonesty, including cheating, plagiarism or sabotage, will be dealt with severely, and disruptive talking and other distractions will not be tolerated. See Student Academic Regulations at http://registrar.mst.edu/academicregs.

Title IX

The title IX policies, resources and reporting options are available online at http://titleix.mst.edu.